The Measurement of Dispersive Effects Using the Mariner 10 S- and X-Band Spacecraft to Station Link

G. A. Madrid
Tracking and Orbit Determination Section

The techniques used to measure dispersive effects from radio metric observables taken at S- and X-band frequencies during the Mariner Venus/Mercury 1973 mission are described. A derivation of the phase and group delay effects is presented based on basic electromagnetic propagation principles and the communication link configuration between the spacecraft and DSS 14.

I. Introduction

The experimental use of an S- and X-band communications link between the Mariner 10 spacecraft and the ground receiver at DSS 14 permitted the measurement of dispersive effects due to charged particles in the transmission media. The dual-frequency capability was made possible by the installation of an experimental receiver developed by G. Levy, et al., at DSS 14 and by the inclusion of an X-band transponder on the spacecraft to complement the operational S-band equipment. Figure 1 illustrates the basic communications interface between the spacecraft and the station: a 2100-MHz uplink received at the spacecraft is retransmitted at S-band frequencies after multiplication by a factor of 240/221; the same signal is retransmitted from the spacecraft at X-band frequencies after multiplication by a factor of 880/221 (or a factor of 11/3 of the S-band retransmission frequency). Dual-frequency techniques for measuring the electron content along the station–spacecraft line of sight were used during the Pioneer 6 and 7 missions and reported by Koehler of the Stanford Electronics Laboratory (Ref. 1). The major difference between the Stanford experiment and the one performed with Mariner 10 is that, in the former, a dual-frequency uplink was used with the results telemetered back to the tracking station, while, in the latter, a dual-frequency downlink was used, providing the capability of detecting both phase velocity and group velocity changes on the doppler and ranging channels, respectively.

The basic principle used in the dual-frequency technique is that electromagnetic propagation in a dispersive medium, by definition, is frequency dependent. In the transmission of a signal at S- and X-bands, however, the frequency separation is such that, upon differencing, the tropospheric dispersive effects remain negligible[1] while

[1]The maximum dispersive effects due to troposphere correspond to a phase path change of about 1 cm at the observers' zenith.

the charged-particle effects remain distinguishable and may be computed. The dispersive effects of charged particles on the downlink phase and group velocities can be determined from the basic characteristics of electromagnetic propagation in a dispersive medium (Ref. 2). The change in phase path length induced by charged particles is given by

$$\Delta \ell = -\frac{b}{w^2}\int_1^3 N\,d\ell \quad \text{(meters)}$$

$$\doteq -\frac{Q}{f^2} I \tag{1}$$

where

N = local electron content density

I = integrated content along the ray path, i.e., $\int_1^3 N\,d\ell$

f = frequency of the transmitted signal

w = radian frequency, $2\pi f$

$b = e^2/2\epsilon_0 M$

$Q = b/4\pi^2 \cong 40.3$ in mks units

e = charge on an electron

m = mass of an electron

ϵ_0 = electric permittivity of free space

The group path delay is merely $-\Delta\ell$, due to the fact that the phase and group effects are equal and opposite. The phase path delay can usually be used only to obtain the charged-particle content relative to the first point of comparison, while group delay provides an absolute measure of the electron content because a point of reference can be found at that first point. By differencing the S- and X-band doppler and range observables from DSS 14, a measure of the dispersive phase and group effects was obtained. The use of these dispersive effects to calibrate the radio metric observables is reported by Winn and Yip in Ref. 3 and in the following article of this volume.[2] Additionally, a measure of the charged-particle content along the ray path was extracted using the relationship stated in Eq. (1) and the information provided to the Mariner Venus/Mercury 1973 Radio Science Team for their examination. A derivation and description of the algorithms used to process the doppler and range observables for the dispersive effects follows.

[2]F. B. Winn and K. B. Yip, "DSN–MVM'73 S/X Dual-Frequency Doppler Demonstration."

II. Dispersive Phase Effects

Referring to Fig. 1, the relationship required to measure the dispersive effects in phase velocity may be derived from the received doppler frequencies as follows:

For S-band, let

$$S_1 = K_1 \cdot f_{os}$$

$$S_2 = S_1\left(1 - \frac{\dot\rho_1}{c}\right) + \Delta S_{1E}$$

$$S_3 = \frac{240}{221} \cdot S_2$$

$$S_4 = S_3\left(1 - \frac{\dot\rho_2}{c}\right) + \Delta S_{2E}$$

$$= K_1 \cdot \frac{240}{221} \cdot f_{os}\left(1 - \frac{\dot\rho_1}{c}\right)\left(1 - \frac{\dot\rho_2}{c}\right) + \frac{240}{221} \cdot \Delta S_{1E}\left(1 - \frac{\dot\rho_2}{c}\right) + \Delta S_{2E}$$

$$S_5 = K_1 \cdot \frac{240}{221} \cdot f_{os} - S_4 + f_b \tag{2}$$

and, for X-band,

$$X_3 = \frac{240}{221} \cdot \frac{11}{3} \cdot S_2$$

$$X_4 = X_3\left(1 - \frac{\dot\rho_2}{c}\right) + \Delta X_{2E}$$

$$= K_1 \cdot \frac{240}{221} \cdot \frac{11}{3} \cdot f_{os}\left(1 - \frac{\dot\rho_1}{c}\right)\left(1 - \frac{\dot\rho_2}{c}\right) + \frac{240}{221} \cdot \Delta S_{1E}\left(1 - \frac{\dot\rho_2}{c}\right) + \Delta X_{2E}$$

$$X_5 = K_1 \cdot \frac{240}{221} \cdot \frac{11}{3} \cdot f_{os} - X_4 + f_b \tag{3}$$

where

S_i = S-band frequencies at points i in link shown in Fig. 1

X_i = X-band frequencies also at points indicated in Fig. 1

ΔS_{1E} = frequency change due to charged particles on uplink

$\Delta S_{2E}, \Delta X_{2E}$ = frequency change due to charged particles on downlink

f_{os} = station oscillator reference frequency, ~44 MHz for block IV receiver

f_b = bias frequency, 5 MHz for block IV receiver

ρ_1 = spacecraft topocentric velocity on uplink

ρ_2 = spacecraft topocentric velocity on downlink

c = velocity of electromagnetic propagation in vacuum

$K_1 \doteq$ uplink multiplier: 48 when $f_{os} \cong 44$ MHz; 96 when $f_{os} \cong 22$ MHz

Then, by subtracting f_b from (2) and (3), factoring 11/3 from (3), and subtracting the results, we find that only the downlink dispersive effects remain, since

$$\frac{(X_5 - f_b)}{\frac{11}{3}} - (S_5 - f_b) = S_4 - \frac{X_4}{\frac{11}{3}} = \Delta S_{2E} - \frac{\Delta X_{2E}}{\frac{11}{3}}$$

However, it is well known that, due to charged particles, $\Delta f \propto 1/f$ (Ref. 2) so that

$$\Delta X_{2E} : \Delta S_{2E} = S : X$$

and, since $X/S = 11/3$, then

$$\Delta X_{2E} = \frac{\Delta S_{2E}}{\frac{11}{3}}$$

Letting $K_X = 11/3$, this means that

$$S_4 - \frac{X_4}{K_X} = \Delta S_{2E} - \frac{\Delta S_{2E}}{K_X^2} = \frac{K_X^2 - 1}{K_X^2} \Delta S_{2E}$$

from which we deduce that

$$\Delta S_{2E} = \left[\frac{K_X^2}{K_X^2 - 1}\right]\left[S_4 - \frac{X_4}{K_X}\right]$$

or, equivalently, that

$$\Delta S_{2E} = \left[\frac{K_X^2}{K_X^2 - 1}\right]\left[\frac{X_5 - f_b}{K_X} - (S_5 - f_b)\right] \qquad (4)$$

Thus relating the downlink dispersive effects in frequency to a difference of the two received doppler frequencies.

III. Dispersive Group Effects

The dispersive effects in group velocity may be derived in a simpler manner by noting that the S-band uplink effects are common to both S- and X-band range measurements, and, since the spacecraft multiplication factor applies only to the carrier and not to the range modulation, any difference between the range measurements at the two frequencies is due entirely to dispersive effects on the downlink, i.e.,

$$\Delta R_{SX}(t) = R_S(t) - R_X(t) = \Delta R_{ES} - \Delta R_{EX}$$

where

$R_S(t)$ = S-band range measurement

$R_X(t)$ = X-band range measurement

ΔR_{ES} = group delay due to charged particles at S-band

ΔR_{EX} = group delay due to charged particles at X-band

Since dispersive path delay is inversely proportional to the square of the received frequency (Ref. 2), then

$$\Delta R_{EX} = \frac{\Delta R_{ES}}{K_X^2}$$

and, therefore,

$$\Delta R_{SX} = \Delta R_{ES}\left(1 - \frac{1}{K_X^2}\right) = \Delta R_{ES}\left(\frac{K_X^2 - 1}{K_X^2}\right)$$

Solving for ΔR_{ES} we then obtain

$$\Delta R_{ES} = \left[\frac{K_X^2}{K_X^2 - 1}\right][R_S(t) - R_X(t)] \qquad (5)$$

thus relating the dispersive group effects to a difference of two simultaneous range measurements taken at two distinct frequencies.

IV. Application

The doppler and range observables obtained from DSS 14 are not in the form such that Eqs. (4) and (5) may be applied directly. Doppler is reported as a cumulative cycle count obtained by measuring the received frequency at predetermined intervals. The contents of the counter are reported at specific sample time τ_c. The cumulative S- and X-band doppler counts may be represented as (see Fig. 1)

$$D_S(t_i) = \sum_{i=1}^{n} \tau_c S_5(t_i)$$

$$D_X(t_i) = \sum_{i=1}^{n} \tau_c X_5(t_i)$$

and the dispersive phase delay equation equivalent to (4) as

$$\Delta\rho_E(t) = \left[\frac{c}{K_1 K_2 f_{os}}\right]\left[\frac{K_X^2}{K_X^2 - 1}\right] \times \left[\frac{[D_X(t) - D_X(t_0) - f_b \cdot (t - t_0)]}{K_X} - [D_S(t) - D_S(t_0) - f_b(t - t_0)]\right] \quad (6)$$

where

$\Delta\rho_E$ = dispersive effect expressed in terms of meters of phase delay relative to t_0

K_1 = ground transmitter multiplier

K_2 = S-band spacecraft transponder multiplier 240/221

K_X = additional spacecraft multiplier required to obtain the downlink X-band frequency, 11/3

$D_S(t), D_X(t)$ = S- and X-band doppler counts accumulated up to time t

$D_S(t_0), D_X(t_0)$ = initial counter readings at time t_0

Similarly, the range equation (5) must be modified in consideration of the fact that the range reported by the sequential ranging machine contains phase velocity effects as well as the theoretically expected group velocity effects[3] (Ref. 4). After adding the appropriate conversion factors to obtain units of meters, the corresponding equation for range is

$$\Delta\rho_E(t) = \left[\frac{c}{64 \cdot K_1 \cdot f_{os}}\right]\left[\frac{K_X^2}{K_X^2 - 1}\right]\left[\frac{R_S(t) - R_X(t)}{2}\right] \quad (7)$$

where $R_S(t)$ and $R_X(t)$ are the range values, in range units, obtained from the R&D ranging machine, which was used in conjunction with the Block IV receiver. Range values from this machine are defined as follows (Ref. 5):

$$R = \left[T - M\frac{128 \cdot 2^N}{3 f_{os}} + \frac{B_{DSN}}{64 \cdot K_1 \cdot f_{os}} + B_{S/C} - Z\right] 64 \cdot K_1 \cdot f_{os}$$

where

T = round-trip light time

M = modulo number

N = number of components

B_{DSN} = ground equipment bias

$B_{S/C}$ = spacecraft bias

Z = zero delay device factor

(The distinction between S and X values corresponds to the carrier frequency which is modulated by the ranging signal.)

Due to anomalous conditions in the experimental receiver hardware, which were corrected as the mission progressed, discontinuities in the doppler data occurred which caused values produced using Eq. (6) to be grossly in error. These problems were ameliorated, however, by merely redefining the interval $(t - t_0)$ to be $(t_i - t_{i-1})$.[4] In this manner each point computed could be evaluated to determine its validity; invalid points were replaced using an estimate of the information lost. A cumulative effect was then produced by summing the incremental points computed over the tracking pass, viz.,

$$\Delta\rho_E(t) = \sum_{i=1}^{n} \widehat{\Delta\rho}_E(t_i)$$

[3]The range measurements produced by the sequential ranging machine are influenced by both phase and group charged-particle effects in an additive manner. The transmitted range modulation is affected in group velocity, while the receiver's synthesized signal is affected by the phase velocity errors inherent in the doppler used for the rate-aided control loop. Differenced range measurements must therefore be divided by 2 to obtain actual dispersive effects.

[4]Based on a suggestion by John Ondrasik, Section 391.

where $\widehat{\Delta\rho}_E(t_i)$ is the incremental phase delay defined over the interval $t_i - t_{i-1}$, and n is the number of points in the pass.

V. Verification

The experience reported by Winn and Yip (Ref. 3 and Footnote 2) provides adequate verification of these techniques. Figure 2 illustrates, however, the generally good comparison between electron content obtained from the dual-frequency doppler technique and the Faraday rotation technique described by Mulhall, et al., in Ref. 6 Naturally, the case shown represents the data obtained on a day when interplanetary plasma effects were negligible.[5] By verifying the accuracy of the technique on cases of this sort, differences between the two data types on other occasions can be attributed to either interplanetary effects or Faraday mapping errors with a high degree of confidence. The exceptional cases occur when, because of hardware problems, the data become inconclusive.

[5]Faraday rotation measurements, taken using the signal from a linearly polarized antenna on an Earth satellite, only provide information regarding the Earth's ionosphere. The dual-frequency techniques described here provide a measure of the change in total columnar content on the downlink path between the spacecraft and station.

References

1. Koehler, R. L., *Interplanetary Electron Content Measured Between Earth and the Pioneer VI and VII Spacecraft Using Radio Propagation Effects*, Report 67-051, Stanford Electronics Laboratories, Stanford University, May 1967.

2. Laurence, R. S., Little, C. G., and Chivers, H. J. A., "A Survey of Ionospheric Effects Upon Earth-Space Radio Propagation," *Proc. IEEE*, Jan. 1964.

3. Winn, F. B., and Yip, K. W., *S/X Dispersive Doppler Demonstration, Report 1*, IOM 391.3-772, Feb. 1, 1974 (JPL internal document).

4. MacDoran, P. F., and Martin, W. L., "DRVID Charged-Particle Measurement With a Binary-Coded Sequential Acquisition Ranging System," in *The Deep Space Network*, Space Programs Summary 37-62, Vol. II, pp. 34–41, Jet Propulsion Laboratory, Pasadena, Calif., Mar. 31, 1970.

5. Chaney, W. D., *Ranging Equations for MVM-73*, IOM 401-3223, June 23, 1972 (JPL internal document).

6. Mulhall, B. D., Ondrasik, V. J., and Thuleen, K. L., "The Ionosphere," in *Tracking System Analytic Calibration Activities for the Mariner Mars 1969 Mission*, Technical Report 32-1499, pp. 45–68, Jet Propulsion Laboratory, Pasadena, Calif., Nov. 15, 1970.

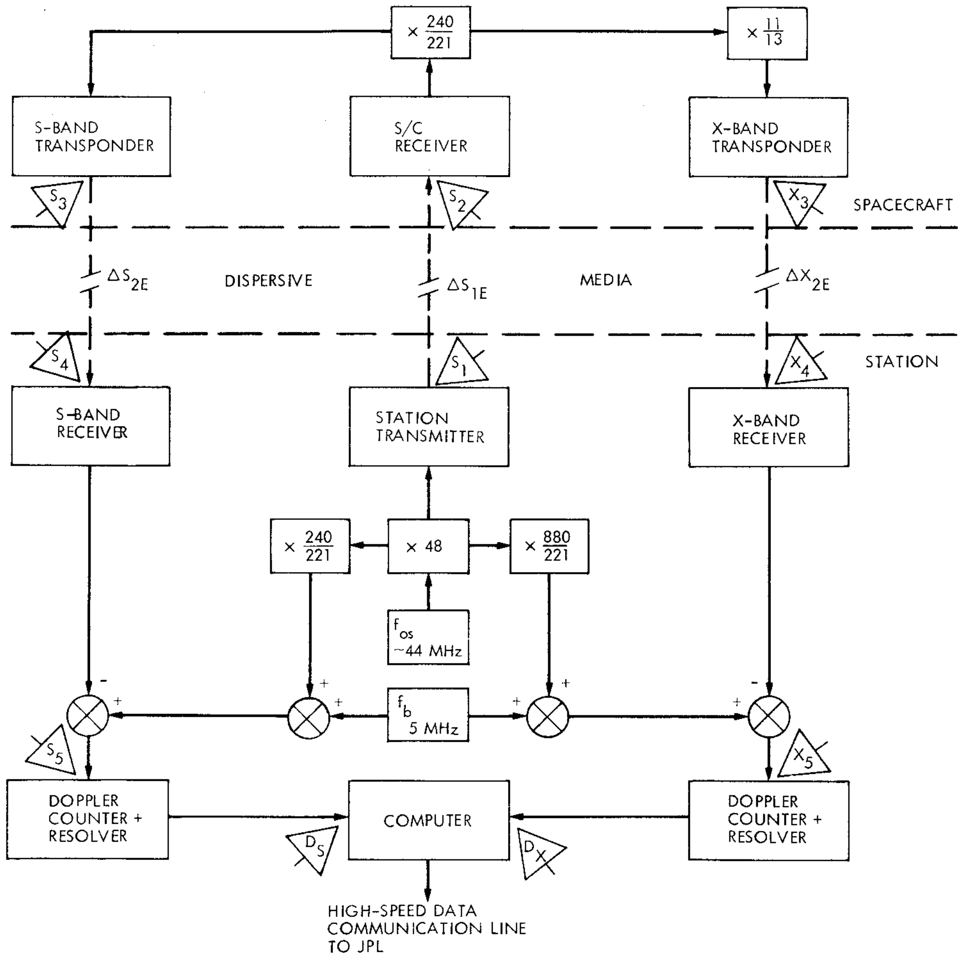

Fig. 1. Conceptual diagram of Mariner 10–DSS 14 dual-frequency doppler configuration

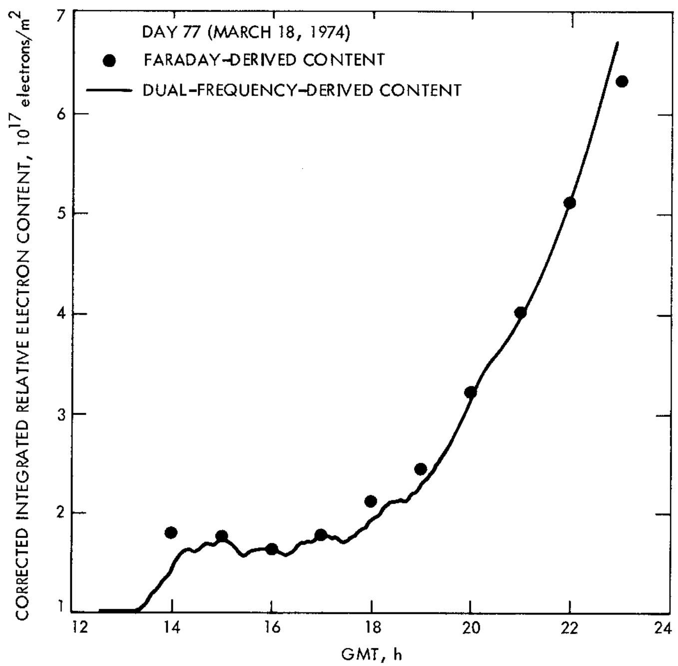

Fig. 2. Comparison of computed dispersive effects and Faraday rotation-derived data on day with apparent negligible plasma activity